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Intelligent risk

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IN THIS ISSUE

EDITOR'S NOTE . . . 2

PODCAST . . . 3

LETTER FROM THE CHAIR . . . 4

CHAPTER EVENTS
Risks in High Frequency Trading . . . 5

Basel II in Turkish Banking and Operational Risk Conference . . . 7

VISIONS OF RISK
On Reverse Stress Testing . . . 8

Sponsored Article — Stressed-out . . . 12

Challenges in Bank Funding and Liquidity: A 3-Part Feature . . . 16

Financial Cartography . . . 20

The Risk of Single Metric . . . 23

CHAPTER REPORTS . . . 24
PRMIA South Africa, London and Brussels

ACADEMIC PARTNER PROFILE . . . 27
Heriot-Watt University and Queen's University

ANNOUNCEMENTS . . . 29
2012 Award Winners

LEARNING OPPORTUNITIES . . . 30

PRMIA LEADERS . . . 32
Board of Directors

ON REVERSE STRESS TESTING

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1. Introduction

Since the recent crisis, stress testing and reverse stress testing have become integral tools in management of banks' balance sheet and capital adequacy. While (forward) stress testing analyzes the impact of stress scenarios on portfolios, reverse stress testing seeks to identify those scenarios that can result in a critical portfolio loss.

Forward stress tests are suitable to assess portfolio sensitivity to specific macro-economic scenarios or compare portfolios of several institutions by using a common set of macro-economic scenarios. In this case, the actual scenario event is the more relevant information than the portfolio loss, as different portfolios, due to their mix, may exhibit different levels of vulnerability to the stress scenario. By contrast, in case of a reverse stress scenario, the critical portfolio loss is the relevant quantity. The aim is identification of those macro-economic scenarios that results in the specified loss level or worse. The critical portfolio loss may be defined by a stress loss expressed as a percentage of the current market value of the portfolio or that of the available capital.

A forward stress scenario can be defined by management or regulator, such as the Federal Reserve Board's Comprehensive Capital Analysis and Review (CCAR), which is based on a set of macroeconomic considerations. Conversely, the definition of reverse stress scenarios requires comprehensive modeling of the scenario space and identification of those scenarios (or clusters of scenarios) that result in a pre-specified critical portfolio loss.

This paper is outlined as follows. In the next section, the notion of a reverse stress test is formalized. Section 3 deals with certain aspects of scenario generation relevant for modeling the scenario space. In section 4 we address some of the practical issues in performing reverse stress testing based on a simple example.

2. Reverse Stress Testing

Assume a portfolio consisting of N positions that are fully described by k time dependent risk factors $X_t = (X_{1,t}, \ldots, X_{k,t})$. The value of position i at time t is denoted by $V_{i,t}$. We assume $V_{i,t}$ to be a function of the risk factors at time t, i.e. $V_{i,t} = V_{i,t}(X_t)$. Note that $V_{i,t}$ may only depend on some risk factors and be independent of the remaining ones. The portfolio value at time t is then given by $V_{pf,t} = \sum_i V_{i,t}$.

We denote the space of the scenarios by Ω. A reverse stress scenario $X_t \in \Omega$ is defined by the equation $V_{pf,t}(X_t) \leq V_{pf,0}(X_0) - CPL$ where $V_{pf,0}$ is the initial portfolio value at time $t = 0$ and CPL denotes the critical portfolio loss. The subspace of reverse stress scenarios with losses that are at least the critical portfolio loss CPL is denoted by Ω_{CPL}.

A modified definition of reverse stress scenarios may be used by introducing a limit of exactly how much worse the portfolio loss is allowed to be (relative to CPL) to avoid inclusion of extreme scenarios. Such a revised definition is given by $V_{pf,0}(X_0) - (1+\alpha\%)CP \leq V_{pf,t}(X_t) \leq V_{pf,0}(X_0) - CP$, where the choice of α sets the degree to which extreme scenarios are considered.

The structure of Ω_{CPL} can be complex and depends on the portfolio. Dividing Ω_{CPL} into clusters can be useful in order to visualize different groups of reverse stress scenarios that correspond to CPL. The more homogenous a portfolio, the less fragmented will be the structure of Ω_{CPL}. Identification of reverse stress scenarios requires knowledge of the scenario space as we do not know in advance which combinations of the risk factors are likely to result in critical portfolio loss. In the next section we deal with scenario generation as an approach to obtain a representative sampling of the scenario space Ω.

3. Scenario Generation

Scenario generation provides the means to obtain a representative sampling of the scenario space Ω. An event X_t in Ω consists of k risk factor values $X_t = (X_{1,t}, \ldots, X_{k,t})$. A risk factor typically belongs to one of the following categories: market risk, credit risk, and macro-economic variables. Examples of risk factors are FX rates, risk-free rates, spreads, rating migration probabilities, GDP, etc. The aim is to model the future development of the risk factors as realistically as possible, which does not necessarily mean best fit to historical data. For example, a low volatility equity market should not suggest keeping volatility low for the entire simulation horizon. Realistic means consistent with historical data and ensuring that simulated values meet the relevant stylized facts. In particular the following widely observed stylized facts should be reproduced: a) absence of autocorrelation for marketable risk factors, b) heavy tails, c) tail dependence, d) gain/loss asymmetry, e) volatility clustering, f) mean reversion, and g) absence of arbitrage opportunities ([1], [2]).

A description of the models used to generate consistent sets of scenarios is outside the scope of this paper. The models are often calibrated based on historical data and a priori information. Historical data is readily available for liquid assets and major macro-economic indicators. However, the available historical data may require validation and preprocessing such as dealing with outliers, different data lengths, data synchronization, or gaps in data. For illiquid assets where historical data is scarce or missing, some proxy data may have to be used. The sample of historical data should be sufficiently long, so as to include several stressed states of markets and economy in order to reflect such states in the calibrations. Note that the choice of historical period can already imply a bias in calibrated model parameters.

In view of stress testing, scenario generation is not only about the modeling of the expected return of risk factors, but also about the distribution of the return values around the expected return. Equally important is a realistic modeling of return co-movements (dependencies) in case of several risk factors. An unbiased estimation of the scenario values, i.e. their distribution, using the historical data often provides an appropriate starting point. *A priori* knowledge can then be introduced to adjust unbiased scenario values, e.g. to match certain expected behaviors.

3.1 GDP Example

To illustrate the ideas presented in the previous section, let us consider U.S. real GDP as example of a macro-economic variable. To simulate future values of the GDP our model was calibrated based on the historical data from March 1991 to June 2009. The start date was selected to include several recent cycles. The choice of the end date will become clear as we proceed. We then generated scenarios for the value of GDP as of June 2010.

Figure 1 shows the distribution of simulated GDP growth values and compares them with the historical growth values observed during the historical data period used for the calibration. Note that the simulated values are unbiased to the extent that only historical data is used to derive them. The distribution of the simulated growth values shows a realistic modeling of GDP growth consistent with the historical values. In particular, the tails of the distribution are heavier than for a normal distribution.

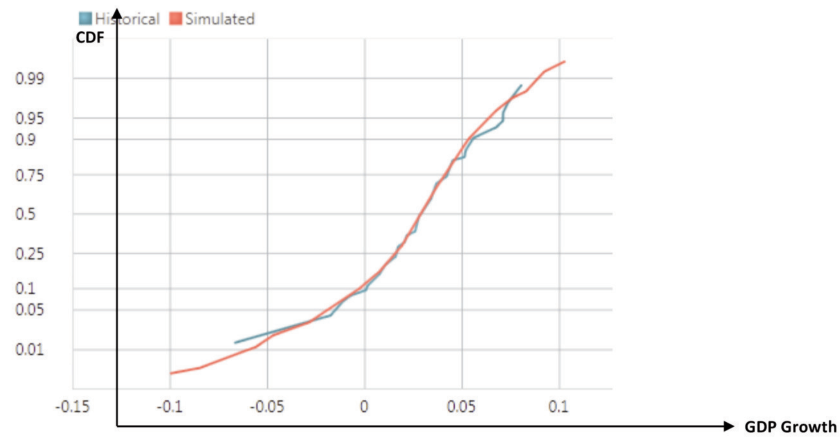

Figure 1: Unbiased Real GDP Growth (U.S., year-over-year 2010), simulated on the basis of data up to June 2009 by an Economic Scenario Generator. Cumulative distributions of simulated and historical growth

A reasonable approach to introduce a bias is to adjust the expected growth value of the simulations. Figure 2 depicts a histogram of year-over-year U.S. real GDP estimates for 2010 based on data available as of June 2009. These estimates originate from 51 institutions contributing to Blue Chip Economic Indicators report [3]. Unbiased estimates using simple auto-regressive models result in values close to the median of the reported forecasts. Given the range of estimates, a choice must be made between the different sources based on trust or a priori information to determine the magnitude of the adjustment that is necessary to match the simulated expected growth to our own a priori expectation. **(Figure 2)**

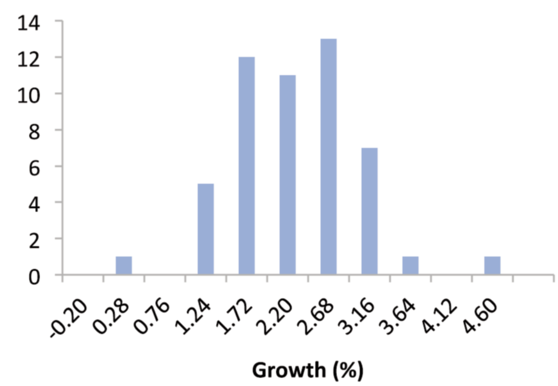

Figure 2: Histogram of Blue Chip Economic Indicators, U.S. Real GDP Forecast, 2010

In **Figure 3**, we have adjusted the simulations downward by 1 percent to reflect our bias on the expected GDP growth. Accordingly, a shifted distribution of simulated values is obtained with a biased expected growth but, consistent with historical data, an unaffected shape of the distribution around the forecast. As a result of the bias, the probability of zero or negative growth increases from 3.5 percent to 11.7 percent.

As final remark, we note that the distribution of unbiased simulated values in Figure 1 is wider than the distribution of estimates provided for the expected growth as shown in **Figure 3**, but not excessively so. This would indicate the degree of uncertainty that comes with a forecast of the expected growth.

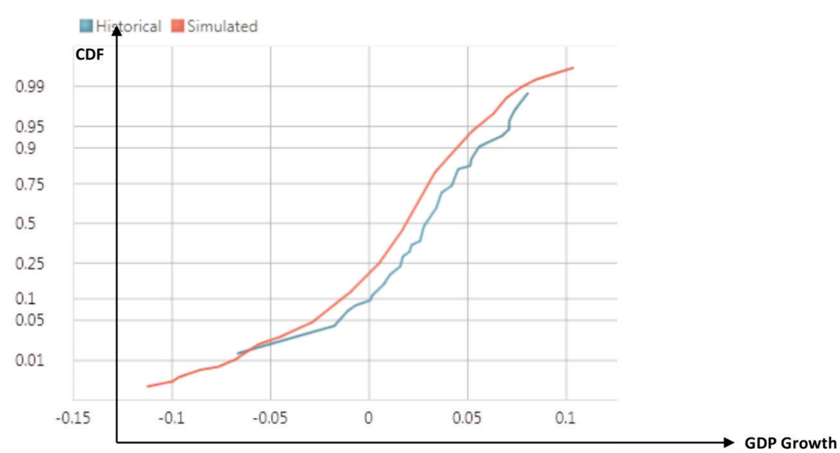

Figure 3: Biased Real GDP Growth (U.S., year-over-year 2010), simulated on the basis of data up to June 2009 by an Economic Scenario Generator. Cumulative distributions of simulated and historical growth. The simulations of Figure 1 have been adjusted to reproduce a given growth expectation value.

4. Reverse Stress Testing

In this section we consider a simple example to illustrate reverse stress testing and draw some conclusions relevant for actual reverse stress tests. We assume a reference portfolio consisting of 5 asset classes as described in Table 1. The example can be generalized to far more complex portfolios including consumer, commercial, and mortgage loan portfolios, commodities, structured products, hedging positions, liabilities, or multi-currency assets.

Asset Type	Description	Allocation Weight
Cash	USD, risk-free	5%
Equity	Dow Jones Total Return	15%
Equity	MSCI UK Total Return	10%
Real-estate	Case-Shiller Index	30%
Risk-free bond	USD, risk--free, MTM 5 Years	40%

Table 1: Reference Portfolio

The valuation currency is chosen to be USD. For this example, the simulated risk factors consist of USD and GBP risk-free rates for maturities 3M, 6M, 1Y, 3Y, 5Y, 7Y, 10Y, 30Y, FX rate GBPUSD, Dow Jones, MSCI UK, Case-Shiller index, U.S. GDP, U.S. CPI, and U.S. unemployment rate. A bond portfolio valuation formula is used to calculate the values of the risk-free bond portfolio with a mean time to maturity of 5 years as a function of the simulated risk-free yields.

We simulated 10,000 stochastic scenarios based on models calibrated on monthly historical data from March 1991 to June 2009. The simulation horizon is 1 year (June 2010). The portfolio was evaluated at this horizon resulting in 10,000 values. We then identified those scenarios that result in a critical portfolio loss CPL (or worse) corresponding to 10 percent of the initial portfolio value, i.e. $V_{pf,1Y}(X_t) \leq 99\% V_{pf,0}(X_0)$. To visualize the space of the resulting reverse stress scenarios, a cluster analysis was performed by applying the k-means clustering algorithm [4]. Using this technique, scenarios are grouped into clusters of scenarios with similar characteristics. The k-means clustering splits the space of scenarios in k clusters $\{C_1, C_2, \ldots, C_k\}$ with centers $\mu = (\mu_1, \mu_2, \ldots, \mu_k)$ such that $\sum_{j=1}^{k}\sum_{X_t^i \in C_j}\left\|X_t^i - \mu_j\right\|^2$ is minimized, where X_t^i is the i-th simulation of risk factors at horizon t, and μ_i is the mean of the i-th cluster.

For the analysis we assumed the number of clusters to be $k = 3$. Given the simplicity of the portfolio and the set of risk factors, this choice is reasonable. In **Figure 4** we have plotted a projection of the clusters to the plane spanned by MSCI UK and Dow Jones equity indices. **Figure 5** depicts a projection to the plane spanned by Case-Shiller index and USD 5 year risk-free rate. We chose the 5-year maturity for projection in order to match the mean time to maturity of the bond portfolio.

Before analyzing the clusters, we note the last historical values of the risk factors as of June 2009: Dow Jones index 9343, MSCI UK 1264, USD 5 year risk-free rate 0.02555, Case-Shiller index 133.19, GBPUSD FX rate 1.6464.

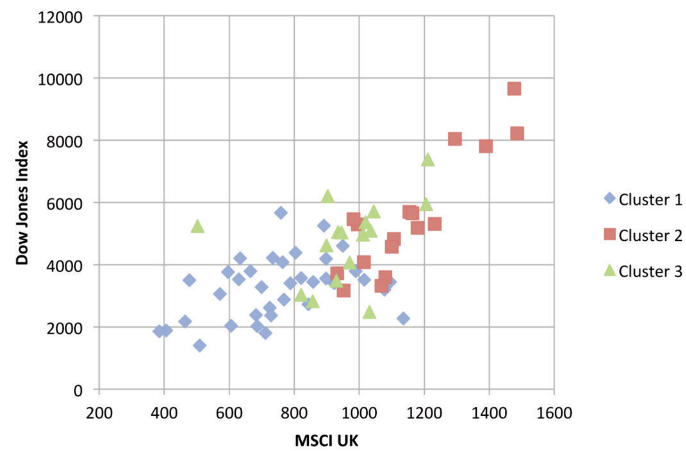

Figure 4: Projection of stress scenarios to the plane spanned by MSCI UK and Dow Jones

As shown in **Figure 4**, Cluster 1 represents scenarios where the portfolio loss is primarily driven by a drop in the equity indices. Clearly the strong dependency of equity markets in stressed conditions (tail dependence) is a major economic factor behind this cluster. By contrast, Cluster 2 is primarily characterized by a decline in Case-Shiller index, see Figure 5. Some of the scenarios in this cluster also show an increase in interest rate. Cluster 3 corresponds to scenarios that are characterized by a strong increase in interest rate, see Figure 5, resulting in a devaluation of the bond portfolio.

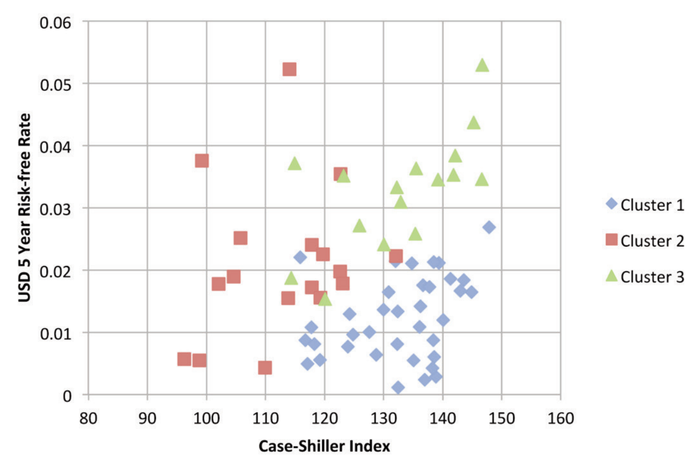

Figure 5: Projection of stress scenarios to the plane spanned by Case-Shiller Index and 5Y risk-free rate

The cluster analysis not only allows us to group the scenarios into clusters with similar characteristics, but also, as a result, we can associate a representative story line to each cluster. Cluster 1 represents an equity shock, Cluster 2 corresponds to a drop in housing prices, and Cluster 3 represents a rise in interest rates (a proxy for a highly inflationary environment).

The example above illustrates a number of points that should be taken into account when performing reverse stress tests. First, it is crucial to use consistent sets of scenarios across different portfolio segments. Scenarios for commercial loan portfolio or market risk portfolio may utilize different sets of risk factors with some common ones; however, they must be consistent with historical observations and any *a priori* information used in view of dependencies and values simulated.

Second, in order to identify reverse stress scenarios one needs to perform a valuation of entire bank portfolio based on a consistent set of scenarios. Identification of reverse stress scenarios based on a stand-alone valuation of constituting sub-portfolios will neglect the dependencies between the sub-portfolios, hence will miss identification of the corresponding scenarios.

Third, a valuation of an actual bank portfolio will involve a substantial number of positions, thereby rendering the identification of reverse stress scenarios computationally challenging. To keep the process computationally manageable, it is constructive to define a stylized portfolio that replicates bulk risks by defining matching portfolios rather than by evaluating individual positions. The reverse stress test is then performed on the stylized portfolio to obtain scenarios that meet the critical portfolio loss criteria. Each cluster can be represented by its center (average of all risk factor values within the cluster). This leads to a reduction of the reverse stress scenarios to a few ones (cluster centers). The resulting cluster centers can then be used for a full-fledged valuation or stress test of the portfolio and a validation of the CPL criteria.

5. Conclusions

A successful reverse stress testing methodology requires a number of ingredients and steps. Realistic stochastic scenarios for all risk factors, preferably produced by an Economic Scenario Generator calibrated using historical data and reproducing stylized facts and dependencies between risk factors is a prerequisite. A comprehensive portfolio description for all assets (and liabilities) of a bank, preferably in the form of a stylized portfolio representing bulk risks can facilitate significantly the process of identifying stress scenarios across all portfolios. This in turn requires valuation formulas quantifying the values of portfolio assets as functions of simulated risk factors. Furthermore, a clustering method can be deployed to analyze and group all the scenarios with losses equal to or larger than the critical portfolio loss. This enables the construction of a few reverse stress scenarios, each representing one of the clusters by averaging the risk factor values per cluster. The obtained stress scenarios can be used for quantitative risk management and (reverse) stress tests as the proposed methodology assigns probabilities to them. Such scenarios are not only quantitative but also qualitative and descriptive as story lines can be developed for the stress scenarios based on the characteristics of the risk factors.

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